# Mathematical Fundamental in Structural Similarity and Pattern on the Plane Complex in Dynamic System

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**Abstract:** The main motivation of this paper is to develop some methods or techniques that will allow us to study complex systems (in the sense of finding their underlying structure or their similarity to others). If we have these techniques, we will be able to tackle a series of real life problems that until now have no reliable solution. Examples of such problems are 3D-object recognition, handwritten word recognition, interpretation of bio-medical signals and speech recognition. In this paper, we will present one technique to analyze dynamic systems based on their behavior, where that behavior can be determined from the system output trajectories. We will use dynamic pattern recognition concepts for dynamic system analysis. This allows us to search for structures in data and classify these structures into categories, so that similarity between structures of the same category is high and similarity between structures of different categories is low.

*Keywords:* Linear Algebra, Dynamic System, Dynamic Pattern Recognition

## INTRODUCTION

Similarity plays a fundamental role in the theories of knowledge and behavior and has been extensively studied in the psychology literature. Traditionally, dynamic system has being studied using formal mathematical theories. However, these approaches to system modeling perform poorly for complex, nonlinear, chaotic, and uncertain system. We believe that a possible way to study and analyze such dynamical system is to restate the problem as a similarity problem.

We ask "is it possible to find a match or similarity between the dynamic system under study and know dynamical system" this approach is motivated by "Case-Based-Reasoning" where the premise is that once a problem has been solved, it is often more efficient to solve a similar problem by starting from the old solution, rather than rerunning all the reasoning that was necessary the first time.

## 2. PROBLEM FORMULATION

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### *Analysis of the Eigenvalues of Linear Dynamic system*

In the following section , we show how the behaviour of any state variable in a linear system can be broken down into different modes of behaviour , each being characterized by an eigenvalue. This paper is concerned with linear system. The temporal trajectory of a state variable $i$

$$x_i(t) = w_{i1}m_1(t) + ... + w_{ij}m_j(t) + ... + w_{in}m_n(t) + u_i \qquad (1)$$

Where $x_i(t)$ is the value of a state variable $i$ in the instant $t$; $w_{ij}$ is a constant term which represents the significance mode $j$ to the variable $i$; $m_{ij}$ is the value of $j^{th}$ mode behaviour in time $t$; $u_i$ is a constant term. The mode of behaviour of a linear system is a function of the eigenvalue of The Jacobian matrix which characterized the system (Oagata 1990)

$$m_j = \begin{cases} \exp\left(\mathrm{Re}\left[\lambda_j\right]t\right) \text{ si } \mathrm{Im}\left[\lambda_j\right]=0 \\ \exp\left(\mathrm{Re}\left[\lambda_j\right]t\right) sen\left(\mathrm{Im}\left[\lambda_j\right]t+\theta\right)=0 \text{ case other} \end{cases} \qquad (2)$$

If the eigenvalues do not have an imaginary part, the part of the behaviour mode is expressed by the first answer of the last equation (2) and is characterized by an exponential growth function if the real part of the eigenvalue is positive and exponential decrease function if the real part of the eigenvalue is negative.

If an eigenvalue has an imaginary part that is different from zero, this means that the two eigenvalues are a conjugated pair (with the same real part) and together they generate the oscillating mode represented by the second expression of the last equation.

If the real part of the conjugated pair of the eigenvalue is positive, an expanded oscillation mode is produced. If it is equal to zero a sustained oscillation mode is produced and if it is negative a dampened oscillation mode is produced. (See figure 1)

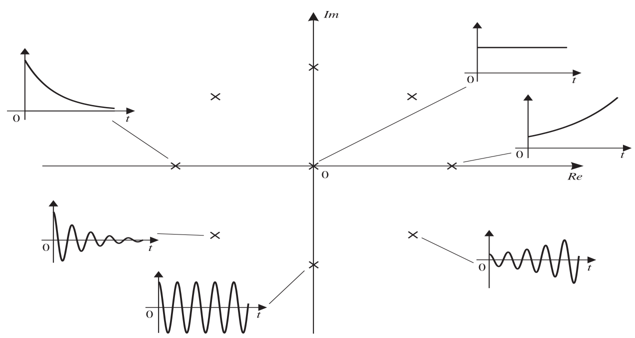


Fig. 1. Plot pattern in el complex plane

The breakdown of the temporary trajectory of a state variable into behaviour modes produces a useful set of diagnostics, not only to understand the sources of behaviour of the variables, but also to identify the degree of interaction between the system variables. Furthermore, the significance of the behaviour mode of variable $w_{ij}$ can also be used as a way of identifying of the elements the structure responsible for the observed behaviour.

## 3. DYNAMIC PATTERN RECOGNITION

Consider a complex system that assumes different states in the course of t time. Each state of the system in the instants of time is considered as object to classify. If a dynamic system is observed temporarily, the variable value of the features constitutes dependent functions of time $[5]$.

The objects receive the name of dynamic whether they represent measurements or observations of a dynamic system and it contains the history of their temporary development. That is to say each dynamic object is a temporary sequence of observations that is described by a discreet function in time. The dependent function of time is represented by a trace, or trajectory, for each object from its initial state to its current state in the space features $[6]$.

The objects B, D, E, G cannot be considered similar and they are divided into the following two groups $\{B,D\}$ and $\{E,G\}$. If the form and orientation of the trajectories are considered irrelevant, their closeness space is then a base for a similarity definition, four clusters are recognized in this way: $\{A,B\}$, $\{C,D\}$, $\{E,F\}$ and $\{G,H\}$

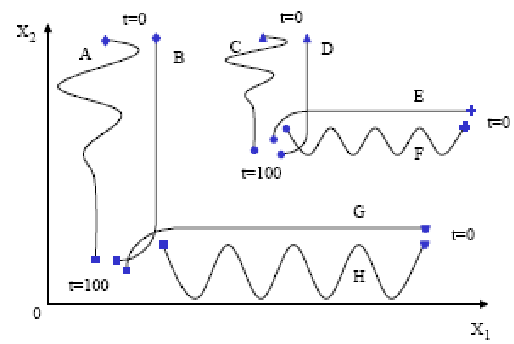


Fig 2 Examples of Structural Similarity on Trajectories

### *3.1 Structural Similarity Based of Slope and Curvature Trajectories*

The curvature of a trajectory at each point describes the grade to which a trajectory is bent in at point. This is evaluated by the coefficient of second derivative of a trajectory in each point that can be defined by the following equation (for one-dimensional trajectory).

$$cv_k = \ddot{x}_k = \frac{\dot{x}_k - \dot{x}_{k-1}}{t_k - t_{k-1}} \quad k = 3,..., p$$

Where $\dot{x}_k$ denotes the coefficient of the first derivative in the point $x_k$ and given for:

$$x_k^{'} = \frac{x_k - x_{k-1}}{t_k - t_{k-1}} \quad \text{k=2,...,p}$$

Substituting the previous equation in the equation of the bend, you arrive to the curvature equation based on the values of the original trajectories

$$cv_k = \ddot{x}_k = \frac{(x_k - 2x_{k-1} + x_{k-2})}{(t_k - t_{k-1})^2} \quad k = 3,..., p$$

## 4. IDENTIFICATION FROM STRUCTURAL SIMILARITY

The transition state of the dynamic system in the internal space and the mapping from the space of internal states to the space of observations is modelled by the following linear equations:

$$x_t = F^{(i)} x_{t-1} + g^{(i)} + w_t^{(i)}$$
$$y_t = H x_{t-1} + v_t$$

Where $F^{(i)}$ is a transition matrix; $g^{(i)}$ is a bias vector. $H$ Is a transition matrix that defines the lineal projection from a space of internal state to the observation space, Notice that each dynamic system has, $F^{(i)}$ $g^{(i)}$ y $w_t^{(i)}$ individually. It is assumed that each $w^{(i)}$ is a noise identifier and $v$ has normal distribution $N_{x_t}\left(0, Q^{(i)}\right)$ and $N_y\left(0, R\right)$ respectively.

The classes of dynamic systems can be categorized by the eigenvalue of the transition matrix which determines the answers of the input zero of the system. In other words, these eigenvalues determine the general behaviour of patterns (trajectories) with temporary variation in the space of states.

The identification of the system with no restrictions is conditioned a temporal ranges $[b, e]$ are represented by the dynamic linear system $D_i$, thus the transition matrix can be estimated $F^{(i)}$ and the bias vector $g^{(i)}$ of the sequence $x_b^{(i)}, \ldots, x_e^{(i)}$ of internal states. This problem of parameter estimation becomes a problem of minimization of error prediction $[5]$.

This error prediction vector can be determined using the discrete equations for dynamical linear systems and after having estimated the matrix of $F^{(i)}$ and bias vector $g^{(i)}$. The formulation is as follows:

$$\varepsilon_t = x_t^{(i)} - \left(F^{(i)} x_{t-1}^{(i)} + g^{(i)}\right) \quad (3)$$

Thus, the sum of the norms of the squares of all of the error vectors in the range $[b, e]$ becomes:

$$\sum_{t=b+1}^{e} \left\| e_t \right\|^2 = \sum_{t=b+1}^{e} \left\| x_t^2 - \left(F^{(i)} x_{t-1}^{(i)} + g^{(i)}\right) \right\|^2 \quad (4)$$

Finally, the optimum values of $F^{(i)}$ and $g^{(i)}$ can be estimated by solving the following problem of least square as:

$$F^{(*i)}, g^{(*i)} = \arg\ \min \min_{F^{(i)}, g^{(i)}} \sum_{t=b+1}^{e} \left\| e_t \right\|^2 \quad (5)$$

## 5. EXPERMENTAL RESULTS

In the following example we implement our technique with the fermentation process, with the specifications of a tank with a volume of 40 litters which contains 25 litters of water. In the bottom of tank, air is injected into water to specific flow rate; the air pressure above water level is controlled by a local mass flow. The state variables in the problem formulation are the measures of the gas flow and the pressure.

Next we illustrate the temporary behaviour of the two state variables mentioned previously:

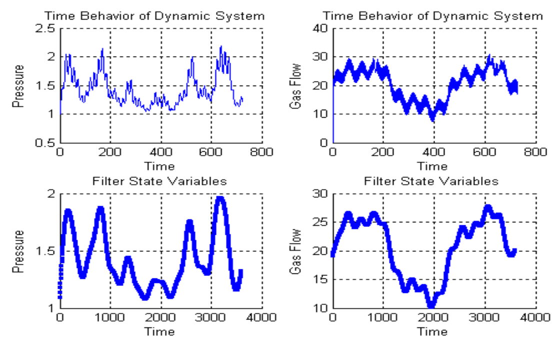


Fig. 3 Dynamic Behavioural of state variable (Fermentation Process)

In the treatment of signs it is necessary to reduce the noise on high grade before start up the stages of the process, such as the identification system using structural similarity. The medium filter is not a technique of digital filtration lineal; it was used to remove noise in the behaviour of the state variables of the fermentation system.

Applying the Castañeda –Colina clustering technique based on the slope and the curvature of the trajectories and labelled in symbolic form by means of colours, we obtain the results that are illustrated in figure 4 where the pressure clusters are show.
We used the method of restricted identification of a system based on eigenvalues, where the transition matrixes were estimated using each segment in the form of an interval and also bias vector. By implementing the theorem of Gershgorin the eigenvalues patterns of the components of the behaviour of the system of fermentation were obtained.

In the previous figure the groups classified by their structural similaridad is illustrated (slope and bend of the trajectories). The curvilinear segments labelled by colours are the implementation of the decomposition of the system in lineal systems by means of similarity, information is also constituted a priori for the process of identification of the system based on structural similarity by means of the estimate of the autovalores, based in the methods of lineal square minimum, the seudoinversa of Penrose and the circles of Gershgorin

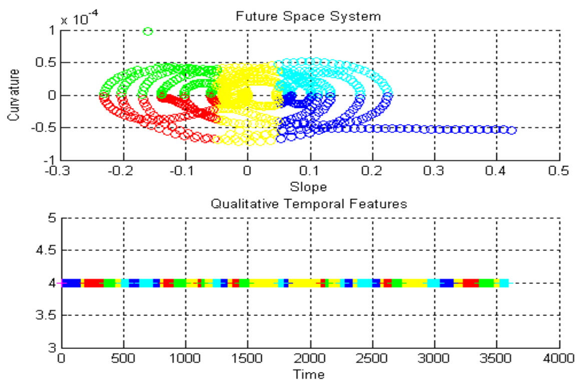


Fig 4 Behaviour in Feature Space and Temporal Space

In figure 5 the clusters in the polar space are also illustrated, as a result of the estimate of the transition matrix and the of the eigenvalues placement. These correspond in this way to the following rules and their symbolic identification:

The eigenvalue patterns of the segments can be observed in figure 5 (left and top). Corresponds to the segment of yellow color and the rule is If $\dot{x}(t)=0$ and $\ddot{x}(t)=0$ then the segment is symbolized with the yellow color . See figure 4

In the top right figure 5 corresponds to the eigenvalue patterns and the rule is if $\dot{x}(t)>0$ and $\ddot{x}(t)<0$ then the segment is symbolized with the blue color. See figure 4

The right bottom figure 5 see eigenvalue pattern corresponds to the segment that satisfies the rule if $\dot{x}(t)<0$ and $\ddot{x}(t)>0$ then the segment is symbolized with the green color . See figure 4

We see the eigenvalue pattern illustrated in figure 5 (Right bottom) corresponds to the segment that satisfies the rule if $\dot{x}(t)>0$ and $\ddot{x}(t)>0$, then the segment is symbolized with the cyan color. See figure 4

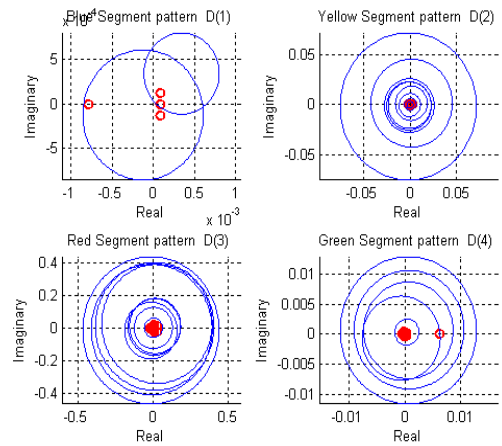


Fig 5 Eingenvalue pattern in Complex Plane

## CONCLUSION

This paper proposed a dynamic clustering from structural similarity to find a set dynamic system which can be implemented for parameter estimation for multiple linear dynamic systems. There are several way to do least square regression , but a linear (in the coefficients) least square ins the simplest , it is know as the Moore Penrose pseudoinverse .

## REFERENCES


[1] Courrieu, P., Fast Computation of Moore-Penrose Inverse Matrices, Neural Information Processing - Letters and Reviews, Vol.8, No.2, August 2005.

[2] Dorf, R.C, Modern Control System, Addison –Wesley, Reading, Mass., 1992.

[3] Föllinger, O., Franke, D., Einführung in die Zustandsbeschreibung dynamischer Systeme. Oldenbourg Verlag, München, 1982

[4] Franze G., Carotenuto, L., Muraca, P. , On the stability of interval matrices, Proceeding of the 2004 American Control Conference, Boston, Massachusetts June 30 -July 2,2004

[5] Kawashima, H., Matsuyama, T. , Proc. 3rd International Conference on Advances in Pattern Recognition (S. Singh et al. (Eds.): ICAPR 2005, LNCS 3686, Springer), pp. 229-238, 2005

[6] Mikenina,Geb, Angstenberger , Larisa, Dynamic Fuzzy Pattern Recognition, Diese Dissertation ist auf den Internetseiten der Hochschulbibliothek online ver,2000

Appendix A. FIRST APPENDIX

En este documento, se define la estimación formal de la matriz de transición $F^{*(i)}$ y el vector de sesgo $g^{*(i)}$ desde la secuencia de estados internos $x_b^{(i)},\dots x_e^{(i)}$ en el rango temporal $\left[b,e\right]$ que es representado mediante $D_i$ sistemas dinámicos lineales. Se introducirá la siguiente notación:

$$X_0^{(i)} \triangleq \left[x_b^{(i)},\dots,x_{e-1}^{(i)}\right],\quad X_1^{(i)} \triangleq \left[x_{b+1}^{(i)},\dots,x_e^{(i)}\right] \quad \text{(B.1)}$$

$$m_0^{(i)} \triangleq \frac{1}{l-1}\sum_{t=b}^{e-1} x_t^{(i)} = \frac{1}{l-1} X_0^{(i)}\left[1,\dots,1\right]^T \quad \text{(B.2)}$$

$$m_1^{(i)} \triangleq \frac{1}{l-1}\sum_{t=b+1}^{e} x_t^{(i)} = \frac{1}{l-1} X_1^{(i)}\left[1,\dots,1\right]^T \quad \text{(B.3)}$$

Donde $l = e-b+1$, así que la sumatoria para la norma cuadrada de todos los vectores errores en el rango viene a ser:

$$\sum_{t=b+1}^{e}\left\|e_t\right\|^2 = \left\|X_1^{(i)} - \left(F^{(i)}X_0^{(i)} + g^{(i)}\left[1,\dots,1\right]^T\right)\right\|^2 \quad \text{(B.4)}$$

$$\begin{aligned}\sum_{t=b+1}^{e}\left\|e_t\right\|^2 &= tr\left(X_1^{(i)} - \left(F^{(i)}X_0^{(i)} + g^{(i)}\left[1,\dots,1\right]^T\right)\right)\\ &\left(X_1^{(i)} - \left(F^{(i)}X_0^{(i)} + g^{(i)}\left[1,\dots,1\right]\right)\right)\end{aligned} \quad \text{(B.5)}$$

$$\begin{aligned}\sum_{t=b+1}^{e}\left\|e_t\right\|^2 &= tr\left(X_1^{(i)}X_1^{(i)}\right) + tr\left(F^{(i)}X_0^{(i)} + g^{(i)}\left[1,\dots,1\right]\right)^T\\ &\left(F^{(i)}X_0^{(i)} + g^{(i)}\left[1,\dots,1\right]\right)\\ &-tr\left(X_1^{(i)}\left(F^{(i)}X_0^{(i)} + g^{(i)}\left[1,\dots,1\right]\right)\right)-\\ &tr\left(\left(F^{(i)}X_0^{(i)} + g^{(i)}\left[1,\dots,1\right]\right)X_1^{(i)}\right)\end{aligned} \quad \text{(B.6)}$$

Usando la identidad derivada de $tr\left(A\right)$ específicamente $tr\left(ABC\right) = tr\left(BCA\right) = tr\left(CBA\right)$ y la propiedad de las transpuestas $\left(AB\right)^T = B^T A^T$ se puede transformar la ecuación anterior así:

$$\begin{aligned}\sum_{t=b+1}^{e}\left\|e_t\right\|^2 &= tr\left(X_1^{(i)}X_1^{(i)}\right) + tr\left(F^{(i)}X_0^{(i)}X_0^{(i)T}F^{(i)T}\right)^T\\ &+tr\left(g^{(i)}g^{(i)T}\right) + \left(l-1\right)tr\left(F^{(i)T}g^{(i)}m_o^{(i)T}\right)+\\ &\left(l-1\right)tr\left(m_0^{(i)}g^{(i)T}F^{(i)}\right) - tr\left(F^{(i)}X_0^{(i)T}X_1^{(i)T}\right)-\\ &\left(l-1\right)tr\left(g^{(i)}m_1^{(i)T}\right) - tr\left(X_1^{(i)}X_0^{(i)T}F^{(i)T}\right) - \left(l-1\right)tr\left(g^{(i)T}m_1^{(i)}\right)\end{aligned} \quad \text{(B.7)}$$

, se diferencia $\sum_{t=b+1}^{e}\left\|e_t\right\|^2$ con respecto a cada $F^{(i)}$ y cada

Si $l-1 \geq n$ (por ejemplo el numero de muestras es igual o mayor que la dimensión de los vectores de estado). Se puede estimar la matriz de transición $F^{*(i)}$ y el vector de sesgo $g^{*(i)}$ mediante la solución del problema de los mínimos cuadrados. Para solucionar este problema de minimización $g^{(i)}$.

$$\frac{\partial}{\partial F^{(i)}}\sum_{t=b+1}^{e}\left\|e_t\right\|^2 T = 2\begin{pmatrix}F^{(i)}X_0^{(i)}X_0^{(i)} + \\ \left(l-1\right)g^{(i)}m_0^{(i)T} - X_1^{(i)}X_0^{(i)T}\end{pmatrix} \quad \text{(B.8)}$$

$$\frac{\partial}{\partial g^{(i)}}\sum_{t=b+1}^{e}\left\|e_t\right\|^2 = 2\left(l-1\right)\left(g^{(i)} + F^{(i)}m_0^{(i)} - m_1^{(i)}\right)$$

Si se asigna cero para todos los elementos diferenciales se obtienen las siguientes ecuaciones :

$$F^{*(i)}X_0^{(i)}X_0^{(i)} + \left(l-1\right)g^{(i)}m_0^{(i)T} - X_1^{(i)}X_0^{(i)T} = 0$$

$$g^{(i)} + F^{(i)}m_0^{(i)} - m_1^{(i)} = 0$$